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QUANTUM REALIZATION OF TERNARY TOFFOLI GATE

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ABSTRACT

Ternary logic synthesis methods have recently been introduced to realize multi-input ternary logic functions using cascades of ternary Toffoli gates. However, realization of ternary Toffoli gates is still very difficult in current quantum technologies. In this paper, quantum realization of ternary Toffoli gate is proposed, which requires lesser gates than the previously published realization.

1. INTRODUCTION

Ternary logic synthesis research has got momentum in the recent years [1-7]. Among them [1, 3, 4] used cascades of ternary Toffoli gates to realize ternary logic functions. The advantage of this approach is that ternary logic functions having many input variables can be easily expressed as Ternary Galois Field Sum of Products (TGFSOP) expression and can be realized using cascades of ternary Toffoli gates [1, 3, 4]. Quantum realization of 1-qudit [1, 4] and 2-qudit gates [7-9] are possible using current quantum technologies. However, in general, any *m*-qudit (*m* > 2) gate is very difficult to realize in quantum technologies, since interaction of more than two particles is nearly impossible to control. Therefore, these gates are realized on the top of 1-qudit and 2-qudit gates. So far as our knowledge is concerned, only one quantum realization of ternary Toffoli gate is proposed on the top of generalized ternary gates (GTGs) [8]. In this paper, we propose another realization of ternary Toffoli gate requiring lesser number of GTGs.

The rest of the paper is organized as follows. In Section 2, we describe some ternary quantum gates. Section 3 presents realization of ternary Toffoli gate using cascade of GTGs. Finally, in Section 4, we conclude the paper.

2. SOME TERNARY QUANTUM GATES

Six 1-qudit (***qu**antum **di**gi**t***) ***ternary Shift gates*** are proposed in [1, 4], which are quantum realizable. Operations and symbols of these gates are shown in Fig. 1, where addition and multiplication operations are over Galois Field 3 (GF3).

Gate Name	Gate Symbol with operaton*
Buffer	x —▷— x
Single-Shift	x —[']▷— $x' = x+1$
Dual-Shift	x —['']▷— $x'' = x+2$
Self-Shift	x —[''']▷— $x''' = 2x$
Self-Single-Shift	x —[#]▷— $x^{\#} = 2x+1$
Self-Dual-Shift	x —[^]▷— $x^{\wedge} = 2x+2$

* Addition and multiplication are over GF3.

Fig. 1 Ternary Shift operations.

Quantum realizable 2-qudit controlled ternary gate family is proposed in [9]. Based on the understanding of paper [9] the 2-qudit ***Generalized Ternary gates (GTGs)*** are proposed in [8] and it is shown in [7] that every GTG can be built from ternary realizable gates from [9]. The GTG is shown in Fig. 2. Here, input *A* is the controlling input and input *B* is the controlled input. The output *P* is equal to the input *A*. The controlling input *A* controls a conceptual ternary multiplexer (a conditional gate) that can be realized using quantum technology such as ion traps [9]. If $A=0$, then the output *Q* is the *x* shift of the input *B*. Similarly, if $A=1$, then the output *Q* is the *y* shift of the input *B* and if $A=2$, then the output *Q* is the *z* shift of the input *B*. Here shift means all ternary shift operations including the Buffer (simple quantum wire). Readers should note that depending on the six possible Shift gates for each of the three positions of *x*, *y*, and *z*, there are $6^3 = 216$ possible GTGs.

Another important controlled ternary gate is 2-qudit ***Feynman gate*** as shown in Fig. 3. Here A is the controlling input and B is the controlled input. The output P is equal to the input A and the output Q is GF3 sum of A and B. Ternary Feynman gate is practically quantum realizable [9].

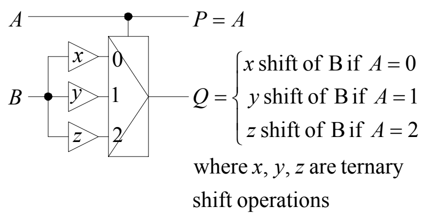

Fig. 2 Generalized Ternary Gate.

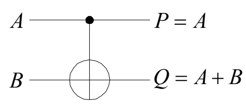

Fig. 3 Ternary Feynman gate.

A very useful controlled 3-qudit ternary gate for multiple-input circuit synthesis is ***Toffoli gate*** as shown in Fig. 4. Design of TGFSOP arrays and factorized arrays is based on these gates [1, 4]. Here the inputs A and B are the controlling inputs and the input C is the controlled input. The output P is equal to the input A, the output Q is equal to the input B, and the output R is equal to $A{\cdot}B + C$, where $\cdot$ and + operators are GF3 multiplication and addition, respectively.

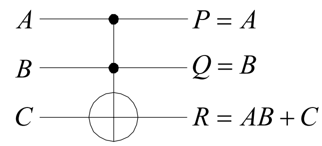

Fig. 4 Ternary Toffoli gate.

3. REALIZATION OF TERNARY TOFFOLI GATE

We give a realization of ternary Toffoli gate using GTGs and ternary Feynman gate in Fig. 5. Fig. 5 is self-explanatory and by verifying the maps of each intermediate signal the reader can verify the correctness of the realization. The first four GTGs are used to realize $A{\cdot}B$. The last four GTGs are the inverse gate of the first five GTGs for restoring the input signals. Finally, $R = A{\cdot}B + C$ is realized using the ternary Feynman gate at the bottom line.

4. CONCLUSIONS

3-qudit ternary Toffoli gate is very important for synthesis of ternary functions having very large number of input variables [1, 4, 5]. Therefore, realization of 3-qudit ternary Toffoli gate on the top of truly realizable 1-qudit and 2-qudit gates is very important. Realization of ternary Toffoli gate using GTGs and ternary Feynman gate is first proposed in [8], which requires 10 GTGs. However, our proposed realization requires only 8 GTGs, which is clearly better than the realization of [8].

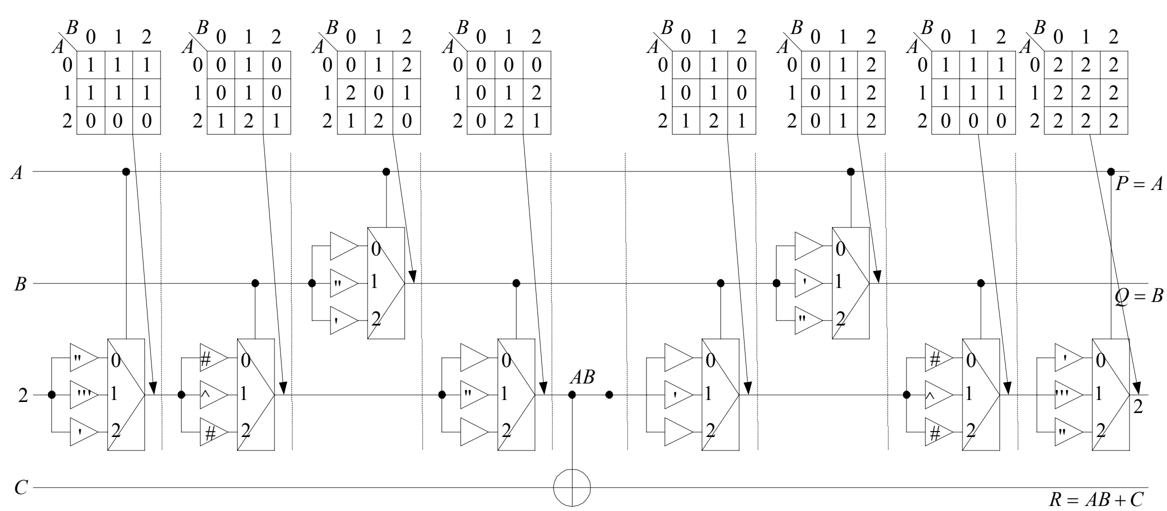

Fig. 5 Quantum realization of ternary Toffoli gate.

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