3rd International Conference on Electrical & Computer Engineering
ICECE 2004, 28-30 December 2004, Dhaka, Bangladesh

QUALITY ASSURANCE ASPECT IN NUMERICAL SIMULATION OF A RANDOM SEA

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ABSTRACT

The simulation of a random seaway requires the generation of representative wave profile realizations of the picked wave spectrum. A uniform distribution U [0, 1] random number generator is sufficient if only the phase of each frequency component is randomized. However, for randomization of both phase and amplitude it is necessary to generate sequences of paired random numbers each with a normal distribution N [0, 1]. The N [0, 1] random numbers are generated from the selected U [0, 1] distribution according to the central limit theorem. The other tasks required for the simulation are the specification of the frequency range, selection of the number of frequency components used to characterize the spectrum realizations simulated, and selection of the regular-frequency or an irregular-frequency time series generation method. The quality of each realization should be judged by the ability of the generation process to maintain the original properties (i.e., spectral moments, location of the maximum spectral ordinate, crossing periods, significant wave height and bandwidth parameter) of the picked spectrum. Objective assessment of this capability together with some clear guidance on how the different statistical decisions influence the quality of the time series generated has been investigated in this paper.

1. INTRODUCTION

In many ocean engineering applications it is of interest to generate time domain realizations of a given random sea state from an experimentally-derived or analytical wave spectrum. Time domain realizations of the wave elevation at a fixed location are often used to calculate the motion behaviour of a moored floating structure. There is a reasonably large literature on the general procedure to generate time realizations as a summation of sinusoidal components from a given wave energy spectrum $S(\omega)$, where ω is the frequency of a component wave. The general procedure is relatively simple and essentially consists of taking the amplitude of each component sinusoid to be equal to $\sqrt{2S(\omega)\Delta\omega}$, where $\Delta\omega$ is the chosen interval of ω. In most cases, it is recommended that the phase angle of each component be chosen as random number uniformly distributed in the interval $(0, 2\pi)$. Descriptions of this procedure are given in [1,2,3]. The direct assignment of wave amplitude from the spectrum ordinates, with only the phase randomly assigned does not represent a random realization unless the number of wave components $N \to \infty$. In this paper, a fully probabilistic approach has been adopted, where both the amplitude and the phase has been randomised. This approach requires the generation of a paired random numbers each with a normal distribution N [0, 1]. The N [0, 1] random numbers are generated from the selected U [0, 1] distribution. Among the other tasks required for the simulation are the selection of a particular sea wave spectrum for which the frequency range, number of wave components and the frequency spacing type (equal or unequal) is to be specified. The quality of the realizations has been examined by comparing the original properties (i.e., spectral moments, location of the maximum spectral ordinate, crossing periods, significant wave height and bandwidth parameter) of the picked spectrum. Finally a general procedure is proposed based upon the above investigations.

2. SIMULATION PROCEDURE

The sea surface is assumed to be Gaussian and the method of simulation is based upon the summation of a finite number of Fourier components to obtain the surface elevation $\zeta(t)$ given by equation (1)

ISBN 984-32-1804-4

$$\zeta(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \varepsilon_i) \qquad (1)$$

where ε_i is the random phase angle of the ith component wave and the amplitude of each sinusoidal component wave is $A_i = \sqrt{2S(\omega_i)\Delta\omega}$, where $S(\omega_i)$ is the input energy spectrum and $\Delta\omega$ is an elemental frequency interval centered on the discrete frequency ω_i. Over the frequency interval $\Delta\omega$ the spectrum $S(\omega_i)$ is assumed uniform. In the summation of the series the number of wave components N , should be as large as possible in order to achieve a correct realization of the random Gaussian process.

Two approaches can be adopted in the selection of the frequency interval $\Delta\omega$; namely equal spacing and variable spacing [4]. In the equal spacing approach the number of frequencies and the selection of the frequency values are determined with regard to the time 'sampling' interval between the quantities numerically predicted and the duration of the simulation. Thus, if the total simulation time is T seconds then the equal spacing frequency interval is given by $\Delta\omega = 2\pi / T$. The simulated wave elevation $\zeta(t)$ will therefore repeat itself after a period of T seconds. The total number of frequency components required in the summation is given by $\omega_1 / \Delta\omega$, where ω_1 is the highest wave frequency for which the input wave spectrum is defined.

The second approach is to use unequally spaced frequency components so that the period of the sine waves are not harmonically related and the series repeats only after a long time. This approach enables much longer simulation times than the equal frequency spacing method for the same number of frequency components. The unequal frequency interval can be chosen in various ways but the most straightforward method is to let $S(\omega)\Delta\omega$ be constant. In other words the amplitude of each component sinusoidal wave is fixed and equal to the constant value σ^2 / N , where N is the number of wave components and σ^2 is the total variance of the time series. This variance equals the area under the wave spectrum.

If the input spectrum is narrow-banded the unequal spacing approach thus has more frequency components within the frequency band where the spectrum has maximum energy. On the other hand the equal frequency interval method distributes the frequency components evenly over the entire frequency range. In this study, the unequal frequency spacing method is adopted where a sequence of paired N [0, 1] random numbers α_i and β_i is generated and a wave spectrum is selected to obtain A_i and ε_i as follows

$$a_i = \alpha_i [S(\omega_i)\Delta\omega]^{1/2} \quad \text{and} \quad b_i = \beta_i [S(\omega_i)\Delta\omega]^{1/2}$$

$$A_i = (a_i^2 + b_i^2)^{1/2} \quad \text{and} \quad \varepsilon_i = \tan^{-1}(b_i / a_i)$$

Now substituting A_i and ε_i in equation (1), the random wave elevation time series can be generated.

3. STATISTICAL DECISIONS

Formal spectral analysis methods allow identification of the form of a sea state spectrum to be derived from the time histories of the water free-surface profile using Discrete Fourier Transforms (DFT) of the original data set. The principal decisions in this process are the selection of the sampling time-interval between discrete measurements and the duration of the recording period such that close peaks are properly isolated, the wave frequency content is properly covered and a balance between resolution and confidence in the spectra is achieved [5]. The total area under the spectrum equals the total variance of the time series and different moments of the spectrum provide different properties such as significant wave height, mean period, crossing period and spectral width parameter.

Fundamental to the generation of random numbers with a specific statistical distribution is the availability of a quality $U[0,1]$ number generator. Wichman & Hill [6] provide such a quality $U[0,1]$ generator. Their algorithm has an extremely large cycle length of 6.95×10^{12}, and to 'iron out' the imperfections of different pseudo-random generators they use the less well-known result that the "fractional part of the sum of n independent rectangular random numbers remains rectangular for all values of n". It is almost a corollary of the CLT, which is concerned with the statistical distribution of the sum rather than the fractional part.

Using the indicated $U[0,1]$ generator the sequences of normally distributed sequence pairs (a_n, b_n) are generated in accordance with the CLT, that is,

$$Z = \left(\sum_{i=1}^{m} N_i - \frac{m}{2} \right) / \left(\sqrt{\frac{m}{12}} \right) \qquad (2)$$

where Z represents N [0, 1] random numbers. Equation (2) can be simplified on the choice of m i.e., how many U [0, 1] random numbers will be added to generate Z. In this paper the investigation is carried out considering $m = 12k$, where k is a positive integer. The normality of the (a_n, b_n) values will be examined using standard χ^2 (chi-squared) based goodness-of-fit statistical tests.

The general form of the spectra to be used in this investigation is given by

$$S(\omega) = A\exp(-B/\omega^4)/\omega^5 \qquad (3)$$

This mathematical form includes the well-known ITTC, ISSC, Bretsneider, Pierson-Moskowitz, Ochi six-parameter spectra etc. Before reporting on the applications studied we now make some observations about the mathematical and numerical properties of the selected spectra form.

The spectral moments m_n of a one-sided spectrum $S(\omega)$ is defined by

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \qquad : n \geq 0$$

The spectral moments m_n : n = 0, 1, 2 & 3 of the selected form, equation (3), can be readily shown to satisfy

$$m_n = A\Gamma(1-\frac{n}{4})/(4B^{1-\frac{n}{4}}) = m_0 B^{\frac{n}{4}}\Gamma(1-\frac{n}{4})$$

using the transformation $t = B/\omega^4$. Here $\Gamma(z)$ is the Gamma (Factorial) function, defined for positive real part of z arguments only. In practice the interval of integration will be $[\omega_l, \omega_u]$ with ω_l, ω_u denoting the lower and upper cut-off frequency respectively.

$$m_n^{finite} = \int_{\omega_l}^{\omega_u} \frac{A}{\omega^{5-n}} \exp(-B/\omega^4) d\omega$$
$$= \frac{1}{\Gamma(1-n/4)} \frac{m_n}{(1-n/4)} \left\{ \left(\frac{B}{\omega_l^4} \right)^{1-n/4} \exp(-\frac{B}{\omega_l^4}) M\left(1, 2-\frac{n}{4}, \frac{B}{\omega_l^4}\right) - \left(\frac{B}{\omega_u^4} \right)^{1-n/4} \exp(-\frac{B}{\omega_u^4}) M\left(1, 2-\frac{n}{4}, \frac{B}{\omega_u^4}\right) \right\}$$

The lower and upper cut-off frequencies are assigned as follows

$$\frac{S(\omega_l)}{S(\omega_{\max})} = \frac{S(\omega_u)}{S(\omega_{\max})} = \alpha\%$$

where $\omega_{\max}$ defines location of maximum value of $S(\omega)$. For the ITTC spectrum $A = 172.8H_{1/3}^2/T_1^4$ and $B = 691.2/T_1^4$, with $H_{1/3} = 4\sqrt{m_0}$ and $T_1 = 2\pi m_0/m_1$. Selecting $H_{1/3} = 7.00$ m and $T_1 = 10.2$ s the parameters A and B are respectively 0.782238 and 0.063856. Hence using the theoretical $(0, \infty)$ integration limits gives $m_0 = 3.062499$, $m_1 = 1.886516$, $m_2 = 1.371679$ and $m_3 = 1.410455$.

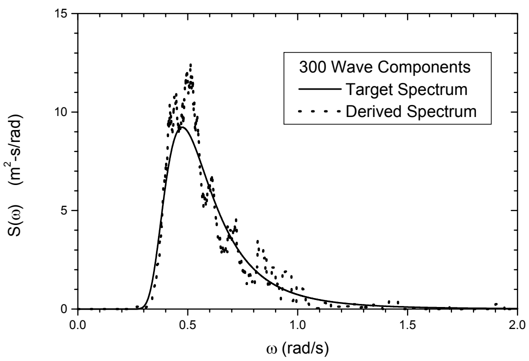
Within the investigations undertaken of the ITTC target spectrum for different $H_{1/3}$ and T_1 values have been generated using α=0.1%, 0.01%, 0.001%, and 0.0001%. The number of frequencies considered are N_w = 50, 100, 200, 300, 400, 500 and 800. The number of $U[0,1]$ values used in the generation of the a_n, b_n values was undertaken for $m = 12k$: k =1, 2, 3, 4, 5, 8 & 12.

The goodness-of-fit of the $U[0,1]$ generated values to the theoretical uniform distribution was examined. The calculated χ^2 values never approached the critical region of the test and may therefore be considered totally satisfactory. Next goodness-of-fit tests were applied to cross-check the normality of the (a_n, b_n) values generated for different m. In each case a realization of 4 hour duration was simulated.

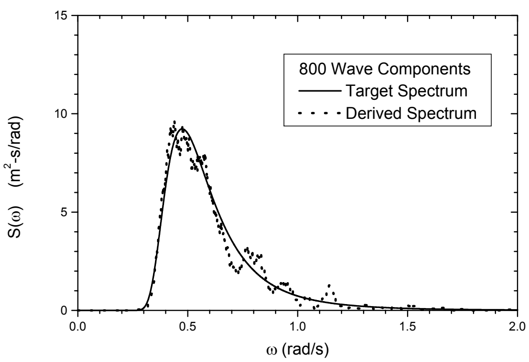
For each derived spectrum the moments m_n : n = 0, 1, 2, 3 & 4 together with $H_{1/3}$, T_1, T_z, T_p and ε were determined. Prior to providing estimates of these quantities the derived spectra can be smoothed by averaging neighboring values within a specified band width. The "window span" used with Tukey-Hamming smoothing is 95.

Space limitations only allow very few results to be presented in this paper. For $\alpha = 0.001\%$ and $k = 8$, a comparison between the target spectrum and the derived spectrum (obtained from the simulated time-series) is presented in Figs. 1a and 1b for $N = 300$ and 800 respectively. Increasing

N is clearly beneficial as revealed from these figures. To establish better confidence in the visually accepted derived spectrum, various spectral properties (i.e., spectral moments, location of the maximum spectral ordinate, crossing periods, significant wave height and bandwidth parameter) as shown in Table 1., need to be computed and compared with the target spectrum results.



(a)



(b)

Fig. 1 Comparison with target and derived spectrum

Table 1: Spectral properties of the derived spectrum

	$N = 50$	$N = 300$	$N = 800$
$\omega_{\max}$	0.530357	0.512024	0.439563
$S(\omega_{\max})$	13.900440	12.502490	9.617363
m_0	3.130754	3.168910	3.026532
m_1	2.018330	1.900384	1.859373
m_2	1.444498	1.282769	1.323502
m_3	1.216443	1.020791	1.163902
m_4	1.418990	1.036238	1.368618
$H_{1/3}$	7.077575	7.120573	6.958773
T_1	9.746231	10.477275	10.227245
ε	0.728228	0.706326	0.759681

It is quite apparent that whereas we cannot be specific about which combination of α, m & N will guarantee realizations with most of the principal characteristics of the target spectrum embedded in the generated time-series, it is possible to indicate how some assessment of the quality assurance of the time-series can be checked. The proposed procedure is as set out below:

A α value of 0.001% is assumed initially to identify (ω_l, ω_u). The theoretical moments m_n and their approximations m_n^{finite} are evaluated using the closed form solutions identified. If m_n and m_n^{finite} are sufficiently close then α is accepted otherwise consider other α values. Now determine how many equally spaced ordinates of $S(\omega)$ are required to provide reasonable m_n^{quad} values, that is, estimates of m_n based on numerical integration (quadrature) satisfying $\left|1 - m_n^{quad} / m_n^{finite}\right| < 5 \times 10^{-5}$. Now choose N at least equal to $N_{ordinate}$ of previous step. Generate time-series for values of k = 4 to 8, and compare properties of derived spectra with m_n^{finite} values and take lower value of k where oscillation occurs in the derived spectra based calculated m_n values. Undertake goodness-of-fit test on the random numbers $U_i[0,1]$, (a_n, b_n) and generated time series to provide confidence in the statistical characteristics of these quantities.

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