

BLOCKING PROBABILITY OF SELF-ROUTING VERTICALLY STACKED OPTICAL BANYAN SWITCHING NETWORKS WITH GIVEN CROSSTALK CONSTRAINTS

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ABSTRACT

To guarantee a high switching speed, routing in vertically stacked optical banyan (VSOB) network needs special attention so that connections are established as fast as possible. Very recently proposed Plane Fixed Routing (PFR) algorithm, which is a distributed control routing algorithm, turns the VSOB networks into self-routing, and therefore provides optimum delay in the order of $\log_2 N$. The PFR algorithm results comparably low blocking probability (<9%) with zero first-order crosstalk. However, the effect of a given amount of crosstalk on the blocking probability has not been evaluated. In this paper, we present the simulation results of the blocking probability of this self-routing VSOB networks with a given crosstalk constraint. Our results show that allowing only a small amount of crosstalk the blocking probability can be reduced to almost zero.

1. INTRODUCTION

Optical networks employing high speed optical switching technologies (such as optical burst switching, optical flow switching or aggressively optical packet switching) are considered promising to meet the demand of emerging high speed and high bandwidth applications, and high-speed optical switch is a critical network element for such optical networks. Directional-coupler (DC) [1] can handle optical signals of some terabits per second and with multiple wavelengths, and this makes it ideal for serving as the basic 2×2 switching element (SE) in high-speed optical switches. It is notable that DC suffers from an intrinsic crosstalk problem [1][2], in which a portion of optical power in one waveguide of a DC will be coupled into the other waveguide unintentionally when two optical flows pass through the DC at the same time.

Banyan networks [3] are attractive for constructing DC-based optical switches for their small depth,

absolute loss uniformity (each path goes through exactly the same number of DCs) and a simple switch setting ability (self-routing). However, as shown in [4], the blocking probability of banyan networks is usually very high. For example, the blocking probability for a 256×256 banyan network can be as high as 0.65. To deal with this situation, it is a novel approach of keeping the whole network nonblocking as well as crosstalk-free by vertical stacking multiple copies (planes) of an optical banyan network [5]. This network is called vertically stacked optical banyan (VSOB) network. However, VSOB lost the self-routing property of the original banyan networks. So far the best known global routing algorithm for crosstalk-free $N \times N$ VSOB network has time complexity in the order of $O(N \log N)$ [6]. For a large size optical switch built on the VSOB structure, this $O(N \log N)$ time complexity is unacceptably long. Instead, a faster routing algorithm is desirable even allowing very low crosstalk and/or blocking probability for a high speed VSOB network. In [7], Khandker et. al. has proposed a fast routing algorithm ($O(\log M)$ time complexity), namely Plane Fixed Routing (PFR) that has single digit of blocking probability (<9%) with zero-first-order crosstalk. PFR actually turns the VSOB into a self-routing network. However, the blocking probability of the network has been evaluated only for zero first-order-crosstalk. In this paper, we present the effect of a given amount of crosstalk on the blocking probability for that self-routing VSOB networks. All logarithms used in this paper are of base 2.

The rest of the paper is organized as follows. We introduce briefly the VSOB network in section 2. In section 3 we present the self-routing VSOB networks along with PFR algorithm. In section 4 presents simulation results for blocking probabilities of the network with a given crosstalk. Also the

hardware cost of the network is presented here. Section 5 concludes the paper.

2. VSOB NETWORK

An $N \times N$ VSOB network is constructed by vertically stacking multiple copies (planes) of an $N \times N$ optical banyan network, with each SE being a DC. The following theorem gives the condition of crosstalk-free and rearrangeably nonblocking VSOB network [6][8].

Theorem: An $N \times N$ VSOB network is rearrangeably nonblocking if and only if its number of planes T meets the following condition:

$$T = 2^{\lfloor (\log N + 1) / 2 \rfloor} \quad (1)$$

Hereafter, we use VSOB (N, T) to refer to the rearrangeably nonblocking $N \times N$ VSOB network with its number of planes T being determined by Eq.1.

The main idea of the routing algorithm is to decompose a permutation evenly into T crosstalk-free realizable partial permutations (CRPPs) and realize each CRPP in a stacked plane of the network. Some input-output hardware is also required. At the input side, N 1: T switches are required to send a signal to only one plane in a routing phase, and at the output side N T :1 combiners are required, but only one of T links of a combiner will be active at a time.

3. SELF-ROUTING VSOB NETWORKS

A switching network is said to be self-routing if each input-signal can determine its path to the destined output by its destination address, regardless of the destination addresses of other connections. In this section first we introduce grouping of inputs and outputs of a banyan network and describe the structure of the self-routing VSOB networks along with the PFR algorithm.

3.1 Grouping of Inputs and Outputs

For an $N \times N$ banyan network, we use the parameter T in Eq.1 to group its input set $I = \{0, 1, \dots, N-1\}$ (output set $O = \{0, 1, \dots, N-1\}$) into the following disjoint groups $I_i (O_i)$ as illustrated in Figure 1.

$$I_i = O_i = \{i.T, i.T + 1, \dots, i.T + (T-1)\}, 0 \leq i \leq \frac{N}{T} - 1 \quad (2)$$

Then we have the following Lemma regarding the property of the inputs grouping in Eq.2 [6][8].

Lemma: Any two connections in an $N \times N$ banyan network will be SE-disjoint from the first stage up to at least the middle stage if their inputs fall within two distinct groups in Eq.2.

The Lemma 1 indicates clearly that if all inputs of a set of connections are group-disjoint, then this set of connections will be crosstalk-free (nonblocking) in at least the first half stages of an optical banyan network. Figure 1(a) shows an example where two connections initiated from different input groups suffer crosstalk at an SE in the second half stages.

It is notable that if all N connections are always evenly distributed among the planes as per Eq. 2, then each plane has only N/T connections, and if we can ensure that each of these N/T connections are initiated from different input groups then, at any instant of time only one input of a group is active; all other inputs are unused. The main idea is that rather than connecting an input to all N/T planes by 1: T switch, we permanently tie an input to a plane. Then all switching elements those never carry active signals are eliminated. Figure 1 explains the idea. Each plane can alternatively be implemented using splitter/SOA gates as shown in Figure 1(b). Splitter/SOA gates for each input will reduce the cost of the switch network.

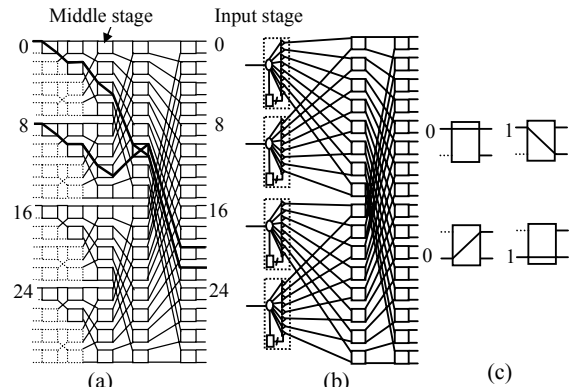


Fig. 1: Grouping of inputs/outputs of a banyan network (a) redundant SEs in dashed lines; thick lines shows connections suffer from one crosstalk SE (b) each plane can also be built using passive splitter/combiner (O) and SOA gates (▷). (c) Routing decision in each SE.

In this case each input control circuit will interpret $\log T$ most significant bits of the address in a header to choose a particular SE in the first stage after the input stage. The header contains the destination address of $\log N$ bits length along with other control bits. So, rest $\log N - \log T$ bits are used for routing through the SEs along the path.

3.2 Plane Fixed Routing (PFR)

In the PFR algorithm, we select one input from each input group and tie them with a plane of VSOB (N, T) network permanently. Since all the inputs tied with a plane are group disjoint, the crosstalk-free

property of Lemma 1 is guaranteed. One possible plane assignment of PFR algorithm can be performed as following. We first define the following subsets of input set $I = \{0,1,\dots, N-1\}$:

$$g_i = \left\{ i, i+T, i+2T, \dots, i + \left(\frac{N}{T}-1\right)T \right\}, 0 \leq i \leq T-1 \quad (3)$$

We then tie all inputs in set g_i to the plane i of the VSOB (N,T) network (suppose the planes are numbered as $0,1,\dots,T-1$). For example, the subsets g_i ($0 \leq i \leq 3$) for VSOB(16,4) network are determined as $g_0 = \{0,4,8,12\}$, $g_1 = \{1,5,9,13\}$, $g_2 = \{2,6,10,14\}$, and $g_3 = \{3,7,11,15\}$, and the final plane assignment of inputs is illustrated in Figure 2.

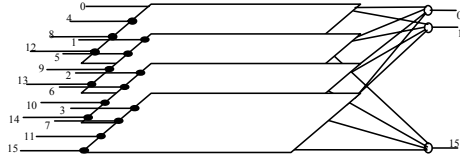


Fig. 2: Plane assignment for inputs in a self-routing VSOB(16,4) network employing PFR algorithm.

Since each plane of VSOB (N,T) is a self-routing banyan network[3],[4] the VSOB (N,T) network employing PFR scheme is actually a self-routing network because there exists a unique path between each input-output pair in such network. Therefore, a VSOB (N,T) network employing PFR scheme has the routing time complexity of $O(\log N)$ that is proportional to number of stages of the network, and this time complexity is optimum because we need at least $O(\log N)$ time to establish a path in a $\log N$ -stage banyan network.

In the PFR algorithm blocking may occur in the last half stages.

4. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the PFR algorithm with different crosstalk constraints in terms of blocking probability and hardware cost.

4.1 Blocking Probability

An extensive simulation has been performed to study the blocking probability, P_b , of self-routing VSOB (N,T) network employing PFR algorithm. The network simulator we developed consists of four major modules as shown in Figure 3.

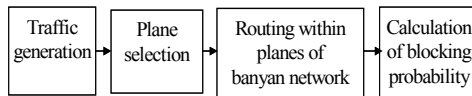


Fig. 3: Block diagram of network simulator.

We consider here the permutation request as the traffic since a permutation does not have output contention, and therefore gives real blocking

probability of the switch network only. Due to the symmetric architecture of VSOB network, every connection request has the same probability to be blocked. In our simulation, we fix the connection request of input-output pair 0-0 and investigate the blocking probability of this connection request only. The traffic generation module randomly generates a permutation request for the self-routing VSOB (N,T) network based on the workload r (here workload r is defined as the occupancy probability of a port). The plane selection module attempts to assign connection requests to different planes using PFR algorithm. Connections assigned to a plane are then established based on the self-routing property of banyan networks. The crosstalk constraint is given in terms number of crosstalk SEs. For example, crosstalk constraint $1 \leq c \leq \log N$ means any connection can have at most c crosstalk SEs along its path. When a 0-0 connection request requires more than c crosstalk SEs to be established, then it is blocked. The blocking probability of a routing algorithm is estimated by the ratio of number of permutation requests, $Perm_{blk}$, in which the 0-0 request is blocked to the total number of permutation requests generated, $Perm_{tot}$; therefore, $P_b = \frac{Perm_{blk}}{Perm_{tot}}$.

4.1.1 Blocking Probability for Self-routing VSOB (N,T) Networks

We have examined four network configurations of $N = \{256, 512, 1024, 2048, 4096\}$. For each configuration, the blocking probabilities are simulated for $r = \{1.0, 0.9, 0.8\}$ and $c = \{0, 1, 2, 3\}$. Graph in Figure 4 shows that the blocking probability reduces to almost zero when crosstalk is only 3 SEs along any connection. The effect of odd and even stages on P_b is also clear here as explained in [7]. Due to space limitation we present only one Table that shows results of the most impressive configuration where number of crosstalk SEs, $c = 3$.

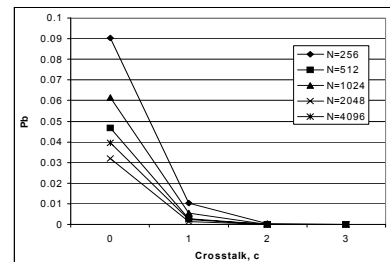


Fig. 4: Blocking probabilities, P_b , of different networks at crosstalk constraint $c = 1$

It is notable that blocking probability decreases sharply with the increase of number of crosstalk SEs.

Table I: Blocking probability with crosstalk constraint $c = 3$

Size, N	Blocking probability		
	r = 1.0	r = 0.9	r = 0.8
256	0.0000040	0.0000020	0.0000020
512	0.0000000	0.0000000	0.0000000
1024	0.0000020	0.0000000	0.0000000
2048	0.0000000	0.0000000	0.0000000
4096	0.0000000	0.0000000	0.0000000

For example, for $N=1024$ and 100% workload blocking probability decreases by 0.0557 with only one crosstalk SE, which is 91% less than that at zero crosstalk SE. However, only 3 crosstalk SEs along any path results almost zero blocking probability for fully loaded switch network as large as 2048×2048 as shown in the Table I.

4.1.2 Hardware cost of Self-routing VSOB(N,T) networks with modification

Each Directional Coupler (or SE) is equivalent to 2 splitters, 2 combiners and 4 SOA gates as shown in Figure 5.

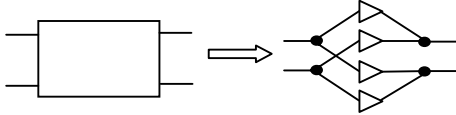


Fig. 5: Equivalent circuit of a directional coupler using splitters/combiners and SOA gates

Each input group is a $T \times T$ network. In the existing self-routing VSOB construction, after eliminating redundant SEs, each group costs $(2^0 + 2^1 + 2^2 + \dots + 2^{\log(T/2)}) = 2^{\log T} - 1 = T - 1$ SEs. These are equivalent to $4(T-1)$ splitters and combiners and $4(T-1)$ SOA gates. That means the T splitters/combiners and T gates in the modified switch construction are almost equivalent to $T/4$ SEs. Counting in this way, the hardware cost of an $N \times N$ modified self-routing VSOB network in terms of SEs is given by Eq.4 and presented in Table II.

$$T_N = \frac{NT}{2} \left(\log \left(\frac{N}{T} \right) + \frac{1}{2} \right). \quad (4)$$

Table II: Comparison among VSOB networks on hardware cost.

Size, N	VSOB, T_N	SOA/Split. VSOB, T_N	Reduction %
256	12032	9216	23.40425532
512	48640	36864	24.21052632
1024	113664	90112	20.72072072
2048	456704	360448	21.07623318
4096	1044480	851968	18.43137255

5. CONCLUSION

In this paper, we have studied the blocking probability of recently proposed self-routing VSOB networks under given crosstalk constraints. We also proposed a modified implementation of the VSOB networks with splitter/SOA gates to reduce the hardware cost. Our simulation shows that only 3 crosstalk SEs along a connection can reduce the blocking probability to almost zero for large switch networks. SOA and splitters/combiners implementation of the switch architecture reduces the hardware cost more than 18%. We believe the results of this paper combined with optimum routing complexity will make the adoption of vertically stacked banyan-based optical switches very attractive in most practical applications. A mathematical model for blocking probability of this network is under research.

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