3rd International Conference on Electrical & Computer Engineering
ICECE 2004, 28-30 December 2004, Dhaka, Bangladesh

ESTIMATION OF AR PARAMETERS AT A VERY LOW SNR USING PREFILTERING AND SUCCESSIVE AUTOCORRELATIONS

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ABSTRACT

This paper comprises a proficient approach for Autoregressive (AR) parameter estimation at a very low signal to noise ratio (SNR) such as -7.5dB. At such a low SNR auto correlated function becomes severely noisy. Spectral estimation is obtained in discrete cosine transform (DCT) domain through filtering of autocorrelation function. Comparing estimated parameters of successive autocorrelations, system poles are identified as the ones that produce minimum deviation.

Key words: DCT, filtering, successive autocorrelation, zero padding.

1. INTRODUCTION

In various signal-processing applications, it is often very effective to represent signals as AR random processes. Such applications include speech analysis, spectral estimation, noise cancellation, econometrics and seismology. Several methods proposed in the literature for parameter estimation use Yule-Walker (YW) equations in the autocorrelation domain [1]-[2]. As noise estimation is not simple, accuracy of the *a priori* noise estimation makes the success of Low Order Yule-Walker (LOYW) equations as well as noise compensated lattice filter (LF) algorithm susceptible to error [3]-[4]. Implementation of High Order Yule-Walker (HOYW) equations does not require *a priori* estimation of noise but it fails to retain accuracy at low SNR as autocorrelation becomes quite noisy due to finiteness of observed data sequence [5]. Hasan and Khan proposed a cosine model to be applied in the autocorrelation domain for system identification at a low SNR [1]. This approach involves preliminary noise estimation based on which a threshold value is obtained. This makes the method vulnerable to noise estimation. A recent technique proposed by Hasan, Fattah and Khan approximates a damped sinusoid in the autocorrelation domain to estimate system pole/pair of poles successively according to their strength at an SNR as low as -5dB [6]. Pre filtering in Fast Fourier Transform (FFT) domain and then application of modified Least Squared HOYW (LSHOYW) equations also successfully identifies system at -5dB SNR [7].

All the recent research works explicitly show that autocorrelation becomes too noisy at very low SNR. For estimation of system parameters with reasonable accuracy noise reduction in autocorrelation domain becomes indispensable. This paper proposes a method that involves filtering in autocorrelation domain by excluding the zeroth lag point. DCT of this function is subjected to appropriate threshold. Inverse DCT (IDCT) provides a noise reduced function from which system poles are identified sequentially according to their strength.

2. PROBLEM FORMULATION

Assuming the AR signal sequence $\{x(n), n=1,2,...\}$ is produced by exciting a zero mean white noise sequence $\{r(n), n=1,2,...\}$ as follows

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + r(n) \qquad (1)$$

where $\{a_k, k=1,2,3,......p\}$ are the system parameters and p is the system order assumed to be known.

If $x(n)$ is contaminated by a white Gaussian noise sequence $v(n)$ of zero mean, the observed signal $y(n)$ can be expressed as

$$y(n) = x(n) + v(n) \qquad (2)$$

Assuming $r(n)$ and $v(n)$ to be uncorrelated we can write $E[r(n)v(n-t)]=0$, for all t, where $E[.]$ denotes expectation operator.

In ideal case the autocorrelation function of output signal $x(n)$ of an all pole (AR) system retains the parameters of the sequence $x(n)$ [8]. The parameters are estimated from YW equations [9]

$$R_{xx}(m) = -\sum_{k=1}^{p} a_x R_{xx}(m-k) \ , \ m \geq 1 \qquad (3)$$

where $R_{xx}(m)$ is the autocorrelation sequence. For positive lags

$$R_{xx}(m) = \frac{1}{N} \sum_{n=0}^{N-1-m} x(n)x(n-m) \qquad (4)$$

where $m \geq 0$ and N is the total number of data points. Autocorrelation of observed noisy signal $y(n)$ can be expressed for positive lags as

$$R_{yy}(m) = \frac{1}{N} \sum_{n=0}^{N-1-m} y(n)y(n-m) \qquad (5)$$

Ideally, for an infinite data sequence $R_{yy}(m)$ should exhibit the following relationship

$$R_{yy}(m) = \begin{cases} R_{xx}(0) + \sigma_v^2 & m = 0 \\ R_{xx}(m) & m > 0 \end{cases} \qquad (6)$$

where σ_v^2 is the noise power. As SNR drops $R_{yy}(m)$ becomes too noisy that no useful information can be extracted without further processing.

The prime objective of this paper is to present an efficient approach for system identification at very low SNR. The method involves filtering of data in autocorrelation as well as DCT domain. The strongest pole/pair of poles is estimated and extracted from autocorrelation. Then remaining poles are estimated comparing minimum deviation of pole positions of successive identifications. The approach is simple and effective for systems even at a very low SNR such as -7.5 dB.

3. FILTERING IN AUTOCORRELATION DOMAIN

Autocorrelation of $y(n)$ over the whole sequence of length N is performed. Response of AR system is decaying but noise is uniformly distributed over the whole sequence in the signal domain. So when autocorrelation is performed, noise effect intensifies and SNR drops with increasing lag. First few positive lags of autocorrelation sustain all the system characteristics and they are less prone to finite data length effect. From (6), it is observed that zeroth lag includes σ_v^2 superimposing on $R_{xx}(0)$. Hence $R_{yy}(0)$, containing significant error, is excluded. The filtered sequence is expressed as

$$\widetilde{R}_{yy}(k) = \begin{cases} R_{yy}(m) & k = m-1 \,\&\, 1 \leq m \leq q \\ 0 & q \leq k \leq N \end{cases} \qquad (7)$$

where $q << N$.

DCT of this zero padded sequence provides highly resolute system spectral distribution. DCT of $\widetilde{R}_{yy}(k)$ can be expressed as

$$Y(l) = c(l) \sum_{n=0}^{N-1} \widetilde{R}_{yy}(k) \cos \frac{\pi(2n+1)l}{2N} \qquad (8)$$

where Y is the array of N transformed values. The coefficients c are given by

$$c(0) = \sqrt{\frac{1}{N}}$$

$$c(l) = \sqrt{\frac{2}{N}} \qquad \text{for} \quad 1 \leq l \leq N-1$$

Spectrum of $R_{yy}(m)$ exhibits a certain noise level whereas $Y(l)$ is noise reduced which is apparent in Fig. 1. In $Y(l)$ there is a slight shift (about 2%) in frequency due to exclusion of $R_{yy}(0)$ that is ignored since $q << N$. As DCT of autocorrelation function should not contain any negative coefficient, negative coefficients of $Y(l)$ are forced to zero [10]. IDCT of this resolute spectrum provides a less noisy estimation, $\widetilde{R}_{1yy}(k)$.

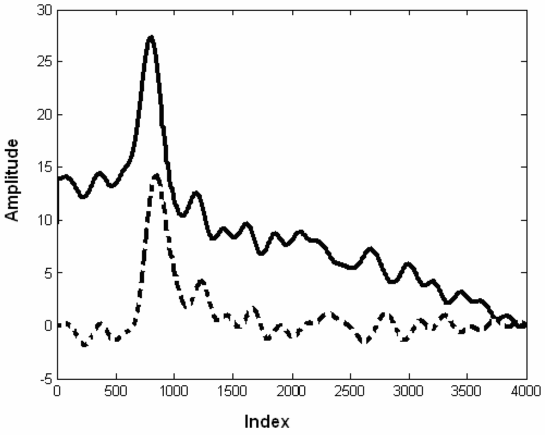

Fig. 1 Spectrum of a 4th order AR system at -7.5 dB
'-' DCT of $R_{yy}(m)$
'…' DCT of $\widetilde{R}_{yy}(k)$

4. PARAMETER ESTIMATION FROM SUCCESSIVE AUTOCORRELATIONS

$\widetilde{R}_{1yy}(k)$ is again auto correlated to obtain further noise reduced function $\widetilde{R}_{2yy}(k)$ and modified LSHOYW equations are applied to first few positive lags except the zeroth one [7]. For better accuracy, system parameters are overestimated i.e. $\widetilde{p}$ system parameters are estimated where $\widetilde{p} > p$, so actual poles as well as some noise peaks are identified. The method proposed in [6] is applied to identify the strongest pole (real)/pair of poles (complex conjugate) in least-squared sense. These identified parameters are filtered out from $\widetilde{R}_{2yy}(k)$ .The filtrate function as expressed in [10],

$$\hat{R}_2(k) = A_g(z)\widetilde{R}_{2yy}(k) \tag{9}$$

where $H_g(z) = \frac{1}{A_g(z)}$ denotes the factor of system function for strongest pole (real)/pair of poles (complex conjugate) in Z-domain.

Although the noise poles are considered in identifying the system poles, unlike the system poles, they do not represent an AR system. Hence, successive autocorrelations of any AR function would retain system poles but noise poles are not expected to remain static. After extraction of strongest pole/pair of poles and successive autocorrelations, positions of system poles in Z-plane remain quite unchanged but noise poles are displaced. So $\hat{R}_2(k)$ is auto correlated to attain $\widetilde{R}_{3yy}(k)$ and parameters are over estimated again. From the first set of estimation, already identified strongest pole parameters are excluded and the rest of the pole positions from two sequential steps are compared in Z-plane. The poles producing minimum deviations are identified as system poles using the knowledge of system order.

5. SIMULATION RESULT

The observation signal $y(n)$, having some predetermined value of SNR is generated by adding a random white Gaussian distribution $v(n)$ to data sequence $x(n)$ for length $N = 4000$ points. $y(n)$ is auto correlated to obtain $R_{yy}(m)$. In (7) q is conveniently chosen to be 40 and $(N-q)$ points are zero padded to obtain a N length filtered sequence $\widetilde{R}_{yy}(k)$. $\widetilde{R}_{1yy}(k)$ and $\widetilde{R}_{2yy}(k)$ functions are generated following the procedure described in section 3 and 4. For a pth order system $(p+2)$ parameters are estimated. In LSHOYW equation matrix, two consecutive equations are formed to make one that gives better results. The strongest pole/pair of poles are obtained as in [6]. This strongest pole/pair of poles is filtered out according to (9). As complete cancellation is not achievable, autocorrelation of $\hat{R}_2(k)$ generates an ARMA function, $\widetilde{R}_{3yy}(k)$ .For lags greater than i , where i represents the length of strongest pole/pair of poles in Z-domain, $\widetilde{R}_{3yy}(k)$ follows AR process. 30 modified LSHOYW equations are applied on $\widetilde{R}_{3yy}(k)$ except first $i = 3$ lags for $(p+2)$ parameter estimation. The remaining pole/pair of poles are identified as described in section 4. We present simulation results for three widely chosen systems by researchers. For system 1 the result is shown in Z-plane in Fig. 2 and for system 2 and 3, estimated values are presented in Table 1a and Table 1b. The effectiveness of this approach is apparent from results. The approach can identify real as well as complex poles and it shows prone to stability.

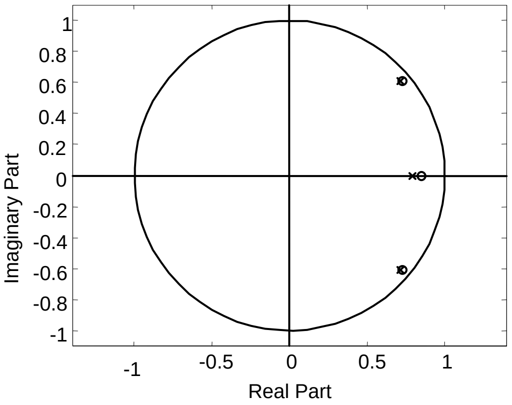

Fig 2: Estimated poles of AR system 1 at SNR-7.5 dB
(o: true, × : estimate)

Table 1a: Estimated AR parameters by the proposed method at SNR =-7.5 dB

	Actual Parameters	Estimated Parameters
System 2	$a_0 = 1$	$a_0 = 1$
	$a_1 =$ -2.7607	$a_1 =$ -2.6670
	$a_2 = 3.8106$	$a_2 = 3.6190$
	$a_3 =$ -2.6535	$a_3 =$ -2.4839
	$a_4 = 0.9238$	$a_4 = 0.8654$

Table 1b: Estimated AR parameters by the proposed method at SNR=-7.5dB

	Actual Parameters	Estimated Parameters
System 3	$a_0 = 1$	$a_0 = 1$
	$a_1 = -1.1$	$a_1 = -1.0976$
	$a_2 = 0.673$	$a_2 = 0.6199$
	$a_3 = -1.0544$	$a_3 = -0.8563$
	$a_4 = 0.9341$	$a_4 = 0.7626$

6. CONCLUSION

This paper presents a simple and proficient approach for system identification with reasonable accuracy at a very low SNR of -7.5dB. Enhanced autocorrelation function is estimated through filtering. Comparisons of over fitted parameters from successive autocorrelations obtain better identification of system poles. At an SNR below -7.5dB autocorrelation function becomes too noisy and spurious noise peaks become significantly stronger than the less dominating system peaks. This makes the technique susceptible to wrong identification.

7. REFERENCES

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