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DISCRITE-TIME SLIDING MODE CONTROL BASED SPEED CONTROL OF INTERIOR PARMANENT MAGNET SYNCHRONOUS MOTOR

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ABSTRACT

This paper presents a discrete-time sliding mode control based speed control of interior permanent magnet synchronous motor (IPMSM). The discrete-time linear model of IPMSM was derived from the view of steady state operating point. The vector control method was used to calculate the steady state quantities. To take the advantageous properties of sliding mode controller, a speed control for IPMSM was designed as which is stable under the variation of load torque and parameters. The performances of the proposed control system were evaluated using computer simulation.

1. INTRODUCTION

The interior permanent magnet synchronous motor (IPMSM) has recently been spread in industrial applications for high-performance application. The IPMSM has been become popular due to the various beneficial features such as high torque to current ratio, large power to weight ratio, high efficiency, high power factor and robustness, etc. The permanent magnet of IPMSM has been embedded inside the rotor so that the rotor is mechanically robust for high speed operation.

In order to obtain high-performance drive system, the properties such as fast and accurate speed response, quick recovery of speed from any change of load torque and parameters are important criteria. These criteria can be achieved by employing the vector control strategy of IPMSM [1]. The vector control of IPMSM is derived by changing the mathematical model from stationary frame to synchronous frame. But when the IPMSM model in the stationary frame is transformed in to synchronous frame, unwanted cross coupling is aroused [2]. Therefore, the speed control of IPMSM was proposed by using conventional PI controller incorporating decoupling control method [2]. The decoupling control of IPMSM in a synchronous frame is depends on the parameters. Moreover, the conventional fixed gain PI controller is very sensitive to step change of reference speed and the variations of load torque and parameters. Unfortunately, stator resistance and permanent magnet flux vary with motor temperature [3]. For this reason, a robust controller is highly desirable for speed control of IPMSM.

In this study, a discrete-time sliding mode control for speed control of IPMSM is proposed. The variable structure system with sliding mode has robust characteristics to parametric uncertainties and external disturbances [4]. To design the proposed sliding mode control (SMC), a discrete-time linear model of IPMSM is derived from the view of steady state operating point. The vector control method is used to calculate the steady state quantities. An augmented system is developed based on the error system and first difference of state quantities. The linear switching function is defined based on the augmented system. The proposed controller is stable under the variations of load torque and parameters. The effectiveness of the proposed controller is verified through the computer simulation.

2. IPMSM DYNAMICS

Fig.1 shows the equivalent d-q axis circuit of IPMSM. According to Fig. 1, the mathematical model of IPMSM can be expressed by the following equations:

$$v_d = R_s i_d + L_d p i_d - \omega_r L_q i_q \quad (1)$$

$$v_q = R_s i_q + L_q p i_q + \omega_r L_d i_d + \omega_r \Psi_a \quad (2)$$

$$T_e = P_n[\Psi_a i_q + (L_d - L_q) i_d i_q] \quad (3)$$

The state equations of IPMSM from (1) ~ (4) can be written as follows:

$$p\omega_r = -\frac{B}{J}\omega_r + \frac{p^2}{J}[\Psi_a i_q + (L_d - L_q) i_d i_q] - \frac{p}{J} T_L \quad (4)$$

$$p i_d = -\frac{R_s}{L_d} i_d + \frac{L_q}{L_d}\omega_r i_q + \frac{1}{L_d} v_d \quad (5)$$

$$p i_q = -\frac{R_s}{L_q} i_q - \frac{L_d}{L_q}\omega_r i_d - \frac{1}{L_q}\omega_r \Psi_a + \frac{1}{L_q} v_q \quad (6)$$

where, v_d, v_q : d- and q-axis stator voltages; i_d, i_q: d- and q-axis stator currents; R_s: stator resistance, L_d, L_q: d- and q-axis inductances, T_e, T_L: electromagnetic and load torques; J: moment of inertia of the motor and load; D: friction coefficient of motor; P_n: number of poles of the motor; ω_r: rotor speed in angular frequency; p= differential operator (d/dt) and Ψ_a: rotor magnetic flux linkage the stator. The linear state equation Eqs. (4)~(6) in a compact form can be written as follows:

$$p\mathbf{x}(t) = \mathbf{A}^s \mathbf{x}(t) + \mathbf{B}^s \mathbf{u}(t) + \mathbf{E}^s d(t) \quad (7)$$

where, States: $\mathbf{x}(t) = [\omega_r \quad i_d \quad i_q]^T$

Inputs: $\mathbf{u}(t) = [v_d \quad v_q]^T$

Disturbance: $d(t) = T_L$,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1/L_d & 0 \\ 0 & 1/L_q \end{bmatrix},$$

$\mathbf{E} = [-p/J \quad 0 \quad 0]^T$, $a_{11} = -(D/J)$,

$a_{12} = (p^2/J)(L_d - L_q) i_q^s$,

$a_{13} = (p^2/J)(\Psi_a + (L_d - L_q) i_d^s)$,

$a_{21} = (L_q/L_d) i_q^s$, $a_{22} = a_{33} = -R_s/L_d$,

$a_{23} = (L_q/L_d)\omega_r^s$, $a_{31} = -L_d(\omega_r i_d^s + \Psi_a)/L_q$,

$a_{32} = -(L_q/L_d)\omega_r^s$.

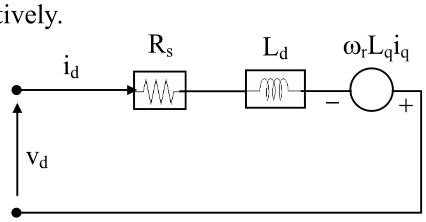
Superscript T and s indicate matrix transpose and steady sate quantities respectively.

The discrete form of (20) can be written as

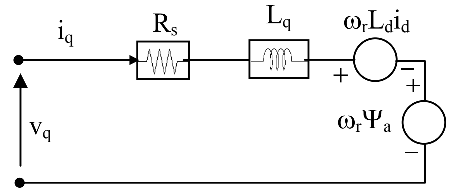
$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{E}d(k) \quad (8)$$

where $\mathbf{A} = e^{\mathbf{A}^s \tau}$, $\mathbf{B} = e^{(\mathbf{A}^s \tau/2)}\mathbf{B}^s$, $\mathbf{E} = e^{(\mathbf{A}^s \tau/2)}\mathbf{E}^s$,

τ and k are sampling period and sampling instant respectively.



(a) d-axis equivalent circuit.



(b) q-axis equivalent circuit.

Fig. 1 Dynamic IPMSM equivalent circuit.

3. CONTROLLER DESIGN

The IPMSM can be operated in both normal and flux-weakening modes. The flux-weakening mode is not included in this study. It is comprehended from Eq. (7) that the speed of IPMSM can be regulated by controlling stator current components v_d and v_q as long as the stator d-axis current i_d is maintained at zero. In order to achieve the desired rotor speed as well as the zero stator d-axis current, two outputs can be defined as follows:

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \quad (9)$$

where, Outputs: $\mathbf{y}(k) = [\omega_r \quad i_d]^T$ and $\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

The reference speed and stator d-axis current is wriiten by

$$\mathbf{R}(k) = [\omega_r^R \quad i_d^R]^T \quad (10)$$

where, superscript R indicates reference value.

According to Eq. (9) and Eq. (10), the error between reference values and actual values can be written as

$$\mathbf{e}(k) = \mathbf{R}(k) - \mathbf{y}(k) \quad (11)$$

An augmented state space that includes the error $\mathbf{e}(k)$ and the incremental state vector $\Delta\mathbf{x}(k)$, and the incremental input $\Delta\mathbf{u}(k)$ can be expressed as

$$\mathbf{X}(k+1) = \mathbf{L}\mathbf{X}(k) + \mathbf{M}\Delta\mathbf{u}(k) \quad (12)$$

where, $\mathbf{X}(k) = \begin{bmatrix} \mathbf{e}(k) \\ \Delta\mathbf{x}(k) \end{bmatrix}$, $\mathbf{L} = \begin{bmatrix} \mathbf{I}_2 & -\mathbf{CA} \\ 0 & \mathbf{A} \end{bmatrix}$, $\mathbf{M} = \begin{bmatrix} -\mathbf{CB} \\ \mathbf{B} \end{bmatrix}$

Eq. (12) is derived with the assumption of that the incremental change of references and disturbance are step function. The objective is to track the desired speed and zero stator d-axis current with zero steady state error in spite of the parameters and load torque variations. The steady state error would be zero provided that the system in Eq. (12) is made stable by the appropriate control input $\Delta\mathbf{u}(k)$. In order to obtain the control input for the stability of augmented system, the discrete-time SMC is applied.

To design discrete-time sliding mode control, linear switching surface $\mathbf{s}(k)$ is considered, which is expressed as follows:

$$\mathbf{s}(k) = \mathbf{GX}(k) \quad (13)$$

In general, the control input in SMC has the following form:

$$\Delta\mathbf{u}(k) = \mathbf{u}_{eq}(k) + \mathbf{u}_{sw}(k) \quad (14)$$

where, $u_{eq}(k)$ is the equivalent control input or nominal for the system motion to be on the sliding surface and $u_{sw}(k)$ is the switching control input which forces the switching function to be zero.

The dynamic of the system in sliding mode is subjected to the constraint $\mathbf{s}(k+1) = \mathbf{s}(k) = 0$ [4] for the sliding mode regime drive. Thus, from the derivation of Eq. (12), the obtained expression for equivalent control input is as follows:

$$\mathbf{u}_{eq}(k) = -[\mathbf{GM}]^{-1}\mathbf{GLX}(k) \quad (15)$$

To imply the controllability of the proposed SMC, **G** is chosen such that $\mathbf{GM} \neq 0$. If we calculate a feedback gain matrix for the augmented system, Eq. (12), the dimensions of the feedback gain matrix and switching function matrix **G** should be same. Hence, the switching function matrix is chosen equal to the feedback gain matrix and where the condition $\mathbf{GM} \neq 0$ is also satisfied. As a result, the switching function matrix **G** (found out by using optimal control theory) is given by

$$\mathbf{G} = -[\mathbf{H} + \mathbf{M}^T\mathbf{PM}]^{-1}\mathbf{M}^T\mathbf{PL} \quad (16)$$

The matrix **P** is the non-negative definite solution of the algebraic Riccati equation

$$\mathbf{P} = \mathbf{Q} + \mathbf{L}^T\mathbf{PL} + \mathbf{L}^T\mathbf{PMG} \quad (17)$$

where, **H** is a positive definite matrix and **Q** is a positive semi-definite matrix.

After stabilizing the system on the sliding surface, the reaching condition has to be forced on the sliding surface to keep on it. With Eq. (13), a Lyapunov function is described as follows:

$$\mathbf{V}(k) = (1/2)\mathbf{s}(k)^2 \quad (18)$$

According to the stability theorem of Lyapunov, the following condition should be satisfied:

$$\Delta\mathbf{V}(k) = \mathbf{s}(k+1)^2 - \mathbf{s}(k)^2 < 0 \quad (19)$$

To satisfy the condition of Eq. (19), the switching control input is chosen as:

$$\mathbf{u}_{sw}(k) = -\eta[\mathbf{GM}]^{-1}\mathbf{GX}(k) \quad (20)$$

where, $0 < \eta < 1$

Since the control law is derived based on Lyapunov stability theorem, the augmented system, Eq. (12), is become stable under the variation of load torque and parameters.

Table I Ratings and Parameters of IPMSM

Rated speed (r/min)	1500
Rated load torque (N-m)	6.0
Number of pole pair	2
Magnetic flux linkage (Wb)	0.533
Stator resistance (Ω)	5.8
d-axis inductance (mH)	44.8
q-axis inductance (mH)	102.7
Inertia ($Kg.m^2$)	0.00039

4. SIMULATION RESULTS

In order to verify the effectiveness of the proposed discrete-time SMC for the speed control of IPMSM, simulations were performed. The rating and parameters of tested IPMSM are shown in Table I. The sampling period and η are chosen 0.5 msec and 0.5, respectively. The matrix values of **H** and **Q** are chosen as follows:

$$\mathbf{Q} = \text{diag}[10.0, 1000.0, 13000.0, 0.0, 0.0]$$

$$\mathbf{H} = \text{diag}[100.0, 2000.0]$$

To obtain good responses the values of **H** and **Q** have been selected by trial and error method. The stated Eq. (4) to Eq. (6) of IPMSM have been solved by using the fourth-order *Runge-Kutta* method.

Fig. 2 shows the transient responses for step change of speed and load torque. In this simulation work, the reference speed is changed from 500 r/min to 1500.0 r/min at 0.5 sec, the load torque is changed from 50% of rated torque to 100% of rated torque at 1.0 sec, and the reference speed is again changed from 1500 r/min to 500.0 r/min at 1.5 sec. Fig. 2(a) shows the response of rotor speed for desired speed. This figure is the evident that the actual speed follows the reference speed without any overshoot under the step change of reference speed and the speed response is stable under the variation of sudden load torque. Fig. 2(b) reveals the transient response of stator current components. It is clear that the stator d-axis current follows the desired zero reference value and the q-axis current is changed due to the change of load torque. It is seen from Fig. 2(c) that the developed electromagnetic torque follows the load torque at steady state. The developed electromagnetic torque is changed due to the changes of speed and load torque at transient condition, which is also reached to the load torque at steady state condition. The variation of load torque is compensated by the developed electromagnetic torque.

Fig. 3 shows the speed response for the variations of stator resistance and q-axis inductance where the reference speed is changed from 500.0 r/min to 600.0 r/min at 0 sec and the parameters are varied at 0.5 sec. Three types of parameter variations are shown in this figure such as stator resistance (R_s) which is increased 100% by its nominal value, q-axis inductance (L_q) which is increased 100% by its nominal value, and the two above mentioned parameters variations are considered at a time. The inner graph of Fig. 3 shows zoomed speed response from 0.4 sec to 0.7 sec. It is cleared in Fig. 3 that the proposed discrete-time is become robust under the variation of parameters of IPMSM.

5. CONCLUSION

A discrete-time SMC is proposed for speed control of IPMSM drive. The proposed controller was designed to work under normal operating condition. Using the proposed control system, the flux-weakening operating condition can also be adapted by changing the reference d-axis stator current which should be calculated according to the algorithm of flux-weakening method for different operating conditions. For the robustness of the proposed controller, the gain matrix was carried out by the Lyapunov's stability criteria. The performances of the proposed controller were verified by computer simulation. The simulation results show good speed response of IPMSM under the variations of load torque and parameters.

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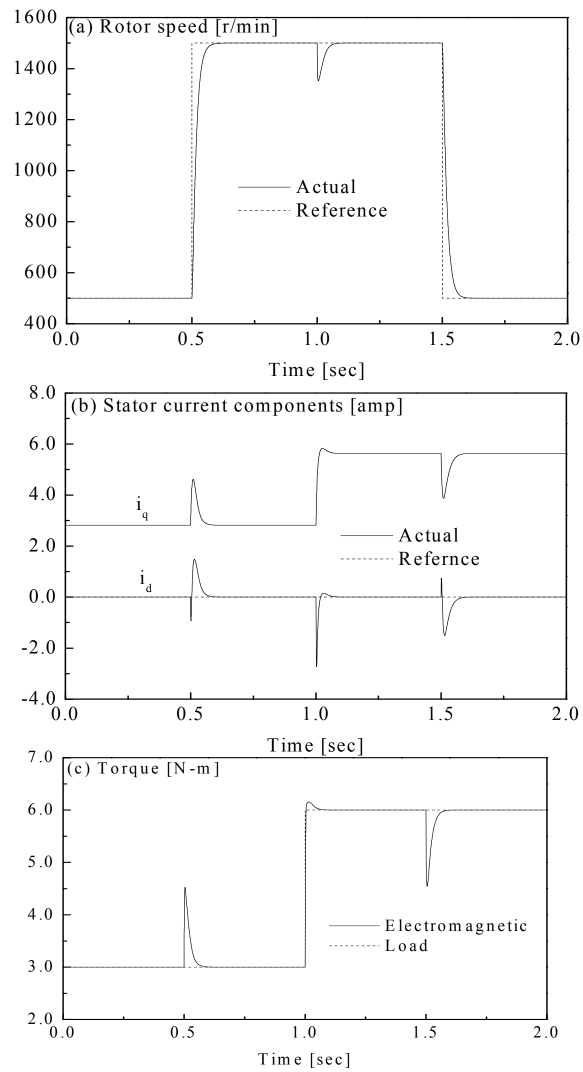

Fig. 2 Response of IPMSM to step change of speed and load torque with the discrete-time SMC.

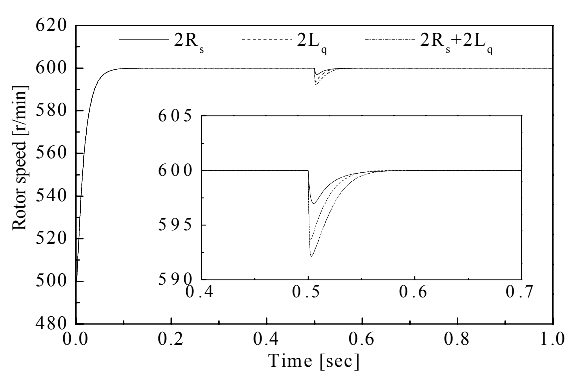

Fig. 3 Response of rotor speed for variations of stator resistance and q-axis inductance with the proposed discrete-time SMC.