CSE100 Lecture01
Computers, Formal Systems, and Strings
Introduction to Computer Systems

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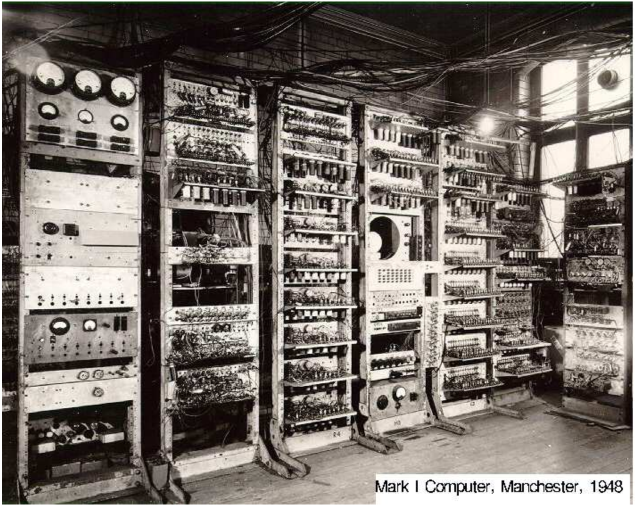
CSE, BUET, 2009

Outline

A Very Early Computer



A Very Recent Computer

Programmable Computers

Informal View

- A computer is a machine that processes given input data and produces desired output data.
- The processing is performed according to a given list of instructions known as a program.
- Programs in a programmable computer are easily changeable as per the need of the user.

Functional View

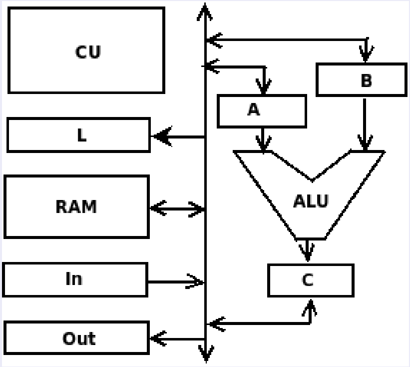
- A computer is a programmable mathematical function f that maps an input x to an output y. For example, $y = f(x)$.

Computer Programs

Compute $z = 2(x + y)$

Input A	;	from keyboard
Store x, A	;	x in memory
Input B	;	from keyboard
Store y, B	;	y in memory
Add	;	$C = A + B$
Store z, C	;	z in memory
Load A, z	;	
Load B, z	;	
Add	;	$C = A + B$
Store z, C	;	
Output C	;	to display

A Simple Computer



Properties and Limitations

Mathematical Properties

- In mathematical terms, all computers are equivalent in the same way that all cars are equivalent.
- Church-Turing Thesis: If you have a problem which can be solved on one computer, then it can be solved on all of them!

Key Limitations

- There are some problems which cannot be solved on any computer, no matter how powerful the computer is.
- There are some functions (even mapping from integers to integers) for which no computer program exists.

Computing Problems

Path Finding Problem

You are in a country having a large number of cities. The cities are interconnected by roads – perhaps one-way and two-way roads both. You are in a given city and you need to go to another. How do you go there? Which route is a minimum-distance route? Which route will have a minimum number of cities in between?

Card Sorting Problem

You have a deck of 52 cards randomly arranged in 4 piles. Only the top card of each pile is visible. You can pick one entire pile and put it on top of another pile or you can just move only the top card from one pile to another. Can you rearrange the deck in a given order? Can you do it using a minimum number of moves?

Formal Systems

Definition

- Change one string to another by using a set of rules.
- Do not go outside the rules – requirement of formality.
- At one time, choose one of the applicable rules freely.
- Rules are one way; a rule cannot be applied backward.
- Formal systems are described by appropriate formal languages.

Observations

- With no experience, you are most likey to violate the rules.
- Playing the game of any formal system is something of an art.

The MIU System

Descritpion

- Given axiom MI, prove theorem MU
- Use only the following inference rules:
  1. $x\mathrm{I} \longrightarrow x\mathrm{IU}$; Append U to a string ending with I
  2. $\mathrm{M}x \longrightarrow \mathrm{M}xx$; Duplicate anything after first M
  3. $x\mathrm{III}y \longrightarrow x\mathrm{U}y$; Replace 3 consecutive I's with U
  4. $x\mathrm{UU}y \longrightarrow xy$; Delete any two consecutive Us

Solution

- Try the mechanical way: produce each string until MU.
- Try the intelligent way: use shortcut, logic, or whatever.

A Formal Proof: MI $\longrightarrow$ MU?

Formulation: Outside The System

- Find I-Count of a string – the number of I.
- Can I-Count of a string be a multiple of 3?
- Generate strings and observe them carefully.

Proof: MU Not Possible from MI

- I-Count starts at 1 as MI is the given string.
- Rules 1 and 4 do not affect the I-Count of a string?
- Rules 2 and 3 do not affect divisibilty by 3.

Inside and Outside A System

Stay Inside

- Follow only the rules of the system, do not break rules.
- You are just a machine, you do not know what you are doing.
- You are not allowed to correlate things to see a broad view.

Jump Outside

- Follow rules little, then observe and think, then find patterns.
- You are intelligent; you are trying to know what you are doing.
- You can think and correlate things freely to see a broad view.

Sets and Strings

Sets

- Sets are collection of things or concepts.
- Members within a set are not in any order.
- {A, B, C, D} is a set of four letters.

Srings

- Strings are sequences of things or concepts.
- Members within a string are in a particular order.
- “BCA” or “2134” are two example strings.

Different Types of Set

Set Size: Number of Elements

- Finite Set: A finite number of elements e.g. $\{1, 2, 3\}$
- Infinite Set: An infinite number of elements e.g. $\{1, 2, 3, \ldots\}$

Infinite Sets: Countability

- Countable Set: Elements can be enumerated by integers e.g. the odd numbers, the even numbers, and the prime numbers.
- Uncountable Set: Elements cannot be enumerated by integers e.g. the real numbers, the strings composed by english letters.

Strings and Symbols

Symbols: A Way for Representation

- We need a symbol to represent each member of a set.
- Symbols are made up of symbols in the form of strings.
- Not all strings are legitimate i.e. valid symbols.

Strings: A Way to Produce Symbols

- Everything is a string of symbols – a book contains letters.
- Each fragment of a string forms a new symbol – words.
- The new symbols are still in a string – a book contains words.

Numbers as Symbols

Symbols for What?

- To represent all kinds of data e.g. a set of shuffled cards.
- To represent all kinds of rule e.g. the card/pile move actions.
- To represent program codes e.g. the card sorting procedure.

Numbers: The Way Forward

- Every thing is represented by just whole numbers.
- Symbols, sets, strings – all are just whole numbers.
- The collection of numbers must be countably infinite.

The MIU System in Numbers

Symbols to Numbers

- M $\iff$ 3, I $\iff$ 1, U $\iff$ 0
- MIIII $\longrightarrow^3$ MUI $\longrightarrow^1$ MUIU $\longrightarrow^2$ MUIUUIU $\longrightarrow^4$ MUIIU
- 31111 $\longrightarrow^3$ 301 $\longrightarrow^1$ 3010 $\longrightarrow^2$ 3010010 $\longrightarrow^4$ 30110

Rules to Numbers

1. xI$\longrightarrow x$IU ;$10m + 1 \longrightarrow 10 \times (10m + 1)$
2. Mx $\longrightarrow$Mxx ;$3 \times 10^m + n \longrightarrow 10^m \times (3 \times 10^m + n) + n$
3. xIIIy $\longrightarrow x$Uy ;$k \times 10^{m+3} + 111 \times 10^m + n \longrightarrow k \times 10^{m+1} + n$
4. xUUy $\longrightarrow xy$;$k \times 10^{m+2} + n \longrightarrow k \times 10^m + n$

Conclusion

Remarks

- Computers are easily-programmable machines.
- Computers represent formal systems only.
- Computers handle just strings of numbers.

References

- EGBGEB: page 33 – 41, 260 – 264.
- www.buet.ac.bd/cse/users/faculty/newton/teaching.html

Questions

Please ask any questions that you might have.